Program V. **Systems of Equations and Inequalities**

SOL Topic:

A.9

The student will solve systems of two linear equations in two variables, both algebraically and graphically, and apply these techniques to solve practical problems. Graphing calculators will be used as both a primary tool of solution and to confirm an algebraic solution.

Activity 1: **Solve the following system graphically, numerically, and confirm analytically.**

$X + 2y = 8$
$2x – y = -12$

Place both equations into slope-intercept form:

- $Y_1 = (-1/2)\, x + 4$
- $Y_2 = 2x + 12$
- Use 2nd CALC, "intersect" to determine where the lines cross.
- The graphical solution is (x, y) = (-3.2, 5.6)
- Examine table values with TblStart = -4 , *d* tbl = .2
- On the table find : $X = -3.2$, $Y_1 = 5.6$, $Y_2 = 5.6$
- Show algebraic solution using linear combination. (x, y) = (-16/5, 28/5)

Activity 2: **Solving practical problems using systems of equations.**

The Video Club of Virginia advertises a membership fee of $20.00 per year and allows its members to rent each video for $1.50 each. Write an equation for the problem: How many videos rentals can you get for $29?

- Let $Y_1 = 20 + 1.5x$
- In Zoom 8, trace to find where Y = 29, Solution is X = 6 videos.
- Show table values.

Video Club of America is a competitor which advertises a membership fee of $25.00 per year and allows its members to rent each video for $1.25 each. Write a linear function for Video Club of America's rental in Y_2.

- $Y_2 = 25 + 1.25x$

Questions:

- When is Video Club of America the least expensive?
- For the memberships to be equal how many videos do you have to rent ?
- Solve the equation **$20 + 1.5x = 25 + 1.25x$.**
- If your parents allow you to rent four videos per month, and this was a Christmas present, on July 31st how much have you spent if you joined Video Club of Virginia? How much have you spent if you joined Video Club of America?

SOL Topic:

AII.11

The student will use matrix multiplication to solve practical problems. Graphing calculators will be used or computer programs with matrix capabilities will be used to find the product.

Activity 3: **Demonstrate weighted averages using Matrix Multiplication.**
A grading system is establishes in a math class with the following criteria:
Tests are 50%, Homework 20%, and Quizzes 30%. The following table gives the average scores for four students in each of the three areas.

Name	Examination	Homework	Presentation
Ellen	71.2	82.86	80
Gene	71.4	81.43	83.33
Mike	69.8	72.9	81.7
Christine	88	84.29	91.67

Place the average scores in matrix A. Place the weights in matrix B.
Multiply [A] [B].

[A] * [B] = [Weighted averages]

$$\underset{[A]}{\begin{matrix} 71.2 & 82.86 & 80 \\ 71.4 & 81.43 & 83.33 \\ 69.8 & 72.9 & 81.7 \\ 88 & 84.29 & 91.67 \end{matrix}} \quad X \quad \underset{[B]}{\begin{matrix} .5 \\ .2 \\ .3 \\ \end{matrix}} \quad = \quad \begin{matrix} 76.172 \\ 76.985 \\ 73.99 \\ 88.359 \end{matrix}$$

- How would you establish, on a scale of 90%-A, 80%-B, 70%-C, 60%-D, and below 60% an F, a grade for Ellen, Gene, Mike and Christine?
- Calculate their grades.

Emphasize that for a class of 30, the first matrix has a dimension of "30 by 3", and the second matrix still has a dimension of "3 by 1".

SOL Topic:

AII.12

The student will represent problem situations with a system of linear equations and solve the system using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.

Activity 4: **Solving systems of equations using matrix multiplication and examining the "reduced row echelon format" on the calculator.**

Use matrices to solve:

$$X - Y + 2Z = 4$$
$$3X + Y - Z = 16$$
$$-2X + 4Y + 3Z = -6$$

- **[A] [X] = [B]**
- **$[A]^{-1}[A][X] = [A]^{-1}[B]$**
- **$[X] = [A]^{-1}[B]$**

Where [A] is the matrix of coefficients, [X] is the matrix of Variables, and [B] is the matrix of constants.

For the Reduced Row Echelon Form use:

- "MATRX", "MATH", "rref ([A])"

SOL Topic:

AII.13

The student will solve systems of linear inequalities and linear programming problems and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.

Activity 5: **Describe use of a linear programming activity.**
A computer manufacturing plant can assemble computer parts using two processes. Hours of unskilled labor, skilled labor, and machine time per computer are given. You can use up to 4200 hours of unskilled labor and up to 2400 hours each of machine time and skilled labor. How many computers should you assemble by each process to obtain a maximum profit?

Process **A** earns a profit of $125 per computer, and Process **B** earns a profit of $160 per computer. 0bjective is to obtain a maximum profit **P.**

P = 125A + 160B

Assembly Hours	Process A	Process B
Unskilled Labor	3	3
Machine Time	1	2
Skilled Labor	2	1

$3A + 3B \leq 4200$ Unskilled Labor
$A + 2B \leq 2400$ Machine Time
$2A + B \leq 2400$ Skilled Labor

Solve each equation for B.

The vertices at the points of intersection of the graph are:

- (0, 1200) P = $192,000
- (400, 1000) P = $210,000 Max Profit
- (1000, 400) P = $189,000
- (1200, 0) P = $150,000
- (0, 0) P = $0

There for the maximum profit is obtained by making 400 component parts using process A and 1000 component parts using process B.

SOL Topic

AII.14

The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.

Activity 5: **Demonstrate a projectile motion problem (quadratic equation) as it relates to a real world situation.**

For a Linear /Quadratic System:

A CO_2 rocket is launched straight up from ground level with an initial velocity ($V_{0)}$ of 150 feet per second. At what time will the rocket be at least 200 feet above the ground? (S_0 =0). What is the maximum height? When will it occur?

The formula for projectile motion is $\mathbf{s = -.5gt^2 + v_0t + s_0,}$ **where** $\mathbf{g = -32ft/sec^2}$

Show Algebraic Solution:

- **-16 x2 + 150x = 200**
- **-16x2 +150x - 200 = 0**

In Function mode:

- Let Y1 = $-16x^2 + 150x$:
- Y2 = 200
- Using 2nd CALC, intersect, compute the two points of intersection which represents time (x-values).
- The solutions are x = 1.6097 seconds and x = 7.7653 seconds.
- The maximum height = 351.56 feet, when x = 4.688 seconds.

Graph in Parametric Mode, Simultaneous Mode:

- Let: X_{1T} = T
- $Y_{1T} = -16T^2 + 150T$
- X_{2T} = 4.688 (Change style of the equation)
- $Y_{2T} = - 16T^2 + 150T$

Window :

- T: [0, 10] $_{.1}$
- X: [-2, 10] $_{1}$
- Y: [-100, 500]$_{100}$